647510 **MSC4MatC403x** **Seat No :_________**

M.Sc.(Maths) Semester - 4 (*CBCS*) Examination
May/June-2021 (NEW COURSE)
Number Theory -2 (CORE)

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q.1(a) If $0 \leq x \leq y, (x, y) = 1$ then show that the fraction $x/_y$ appears in the y^{th} and all later raws. **4 Marks**

Q.1(b) **Answer any two question out of three.** **10 Marks**

(1) If n is a positive integer and x is real then there is a real number $a/_b$ such that $0 < b \leq n$ and $\left|x - \frac{a}{b}\right| \leq \frac{1}{b(n+1)}$. **5**

(2) If a polynomial equation with integral coefficients $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0 = 0, a_n \neq 0$ has a non zero rational solution $a/_b$ where the integers a and b are relatively prime then show that $a/_{a_0}$ and $b/_{a_n}$. **5**

(3) If x and y are positive integers then show that the inequalities $\frac{1}{xy} \geq \frac{1}{\sqrt{5}}\left(\frac{1}{x^2} + \frac{1}{y^2}\right)$ and $\frac{1}{x(x+y)} \geq \frac{1}{\sqrt{5}}\left(\frac{1}{x^2} + \frac{1}{(x+y)^2}\right)$ can't hold **5**

Q.2(a) Show that any rational number can be written as a finite simple continued fraction. **4 Marks**

Q.2(b) **Answer any two question out of three.** **10 Marks**

(1) Expand $\sqrt{5}$ as an infinite simple continued fraction. **5**

(2) Show that the value of any infinite continued fraction is an irrational number. **5**

(3) Expand the rational fraction $19/_{51}$ into finite simple continued fraction. **5**

Q.3(a) Find the irrational number having continued fraction expansion $\langle 8, \overline{1, 16}\rangle$. **4 Marks**

Q.3(b) **Answer any two question out of three.** **10 Marks**

(1) Given any irrational number ξ show that there exist infinitely many rational numbers $h/_k$ such that $\left|\xi - \frac{h}{k}\right| < \frac{1}{\sqrt{5}k^2}$. **5**

(2) If p, q is a positive solution of $x^2 - dy^2 = 1$,then $p/_q$ is a convergent of the continued fraction expnasion of $\sqrt{d}$. **5**

(3) Find all positive solutions of $x^2 - 2y^2 = 1$, for which $y < 250$. **5**

Q.4(a) If x, y, z is a primitive Pythagorean triple then show that one of the integers x or y is even while the other is odd. **4 Marks**

Q.4(b) **Answer any two question out of three.** **10 Marks**

(1) Find all solutions of $10x - 7y = 17$. **5**

(2) Obtain all primitive Pythagorean triples x, y, z in which $x = 40$. **5**

(3) Prove that in a primitive Pythagorean triple x, y, z the product xy is divisible by 12, hence $60/_{xyz}$. **5**

Q.5(a) Prove that $ax + by = a + c$ is solvable if and only if $ax + by = c$ is solvable. **4**

Q.5(b) **Answer any two question out of three.** **10 Marks**

(1) Prove that the radius of the inscribed circle of a Pythagorean tringle is always an integer. **5**

(2) If x_0, y_0 is a positive solution of the equation $x^2 - dy^2 = 1$ then prove that $x_0 > y_0$. **5**

(3) Solve the linear Diophantine equation $172\, x + 20y = 1000$ by means of simple continued fractions. **5**

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